section 3.2

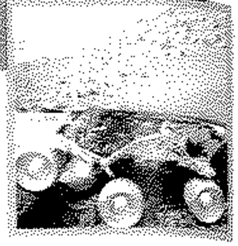
UNCERTAINTY IN MEASUREMENTS

objectives

- Distinguish among the accuracy, precision, and error of a measurement
- Identify the number of significant figures in a measurement and in the result of a calculation

key terms

- accuracy
- precision
- accepted value
- experimental value
- error
- percent error
- significant figures



On July 4, 1997, the Mars Pathfinder spacecraft landed on Mars. Shortly after landing, the spacecraft released a small robotic rover called Sojourner to explore the Martian surface around the landing site in Ares Vallis. NASA scientists had to make thousands of precise calculations to ensure that Pathfinder reached its destination safely. All measurements have some uncertainty. **How do scientists ensure the accuracy and precision of their measurements?**

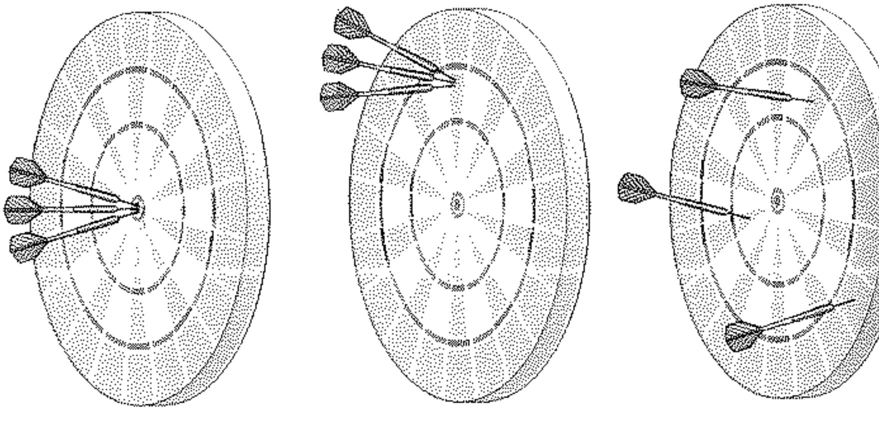
Accuracy, Precision, and Error

Your success in the chemistry lab and in many of your daily activities depends on your ability to make reliable measurements. Ideally, measurements are both correct and reproducible.

Correctness and reproducibility relate to the concepts of accuracy and precision, two words that mean the same thing to many people. In chemistry, however, their meanings are quite different. **Accuracy** is a measure of how close a measurement comes to the actual or true value of whatever is measured. **Precision** is a measure of how close a series of measurements are to one another. Note that the precision of a measurement depends on more than one measurement.

By contrast, an individual measurement may be accurate or inaccurate. Darts on a dartboard illustrate accuracy and precision in measurement. Let the bull's-eye of the dartboard represent the true, or correct, value of what you are measuring. The closeness of a dart to the bull's-eye corresponds to the degree of accuracy. The closer it comes to the bull's-eye, the more accurately the dart was thrown. The closeness of several darts to one another corresponds to the degree of precision. The closer together the darts are, the greater the precision and the reproducibility. Look at **Figure 3.4** as you consider the following outcomes.

(a) All of the darts land close to the bull's-eye and to one another. Closeness to the bull's-eye means that the degree of accuracy is great. Each dart in the bull's-eye corresponds to an accurate measurement of a value. Closeness of the darts to one another indicates high precision.



(a) Good accuracy
Good precision

(b) Poor accuracy
Good precision

(c) Poor accuracy
Poor precision

Figure 3.4

The distribution of darts illustrates the difference between accuracy and precision. ***(a)*** *Good accuracy and good precision: The darts are close to the bull's-eye and to one another.* ***(b)*** *Poor accuracy and good precision: The darts are far from the bull's-eye but close to one another.* ***(c)*** *Poor accuracy and poor precision: The darts are far from the bull's-eye and from one another.*

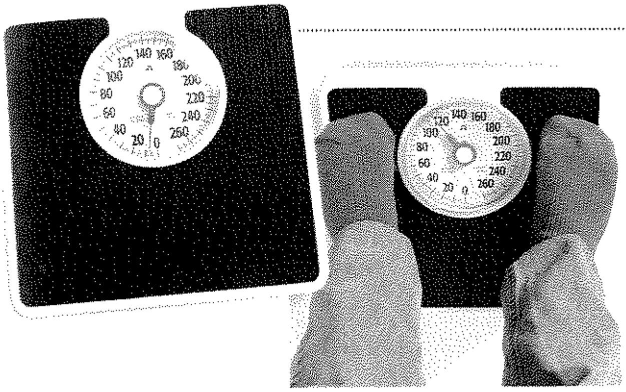


Figure 3.5

This scale has not been properly zeroed. So the reading obtained for the person's weight is inaccurate. There is a difference between the person's correct weight and the measured value.

ll of the darts land close to one another but far from the bull's-eye. he precision is high because of the closeness of grouping and thus the igh level of reproducibility. The results are inaccurate, however, ecause of the distance of the darts from the bull's-eye.

he darts land far from one another and from the bull's-eye. The esults are both inaccurate and imprecise.

o evaluate the accuracy of a measurement, it must be compared with orrect value. Suppose you use a thermometer to measure the boiling of pure water at standard atmospheric pressure. The thermometer 99.1 °C. You probably know that the true or accepted value of the ng point of pure water under these conditions is actually 100.0 °C. e is a difference between the **accepted value,** which is the correct value d on reliable references, and the **experimental value,** the value ured in the lab. The difference between the accepted value and the rimental value is called the **error.**

$$\text{Error} = \text{experimental value} - \text{accepted value}$$

can be positive or negative depending on whether the experimental e is greater than or less than the accepted value.

or the boiling-point measurement, the error is 99.1 °C − 100.0 °C, or °C. The magnitude of the error shows the amount by which the exper- ital value is too high or too low, compared with the accepted value. 1, it is useful to calculate the relative error, or percent error. The **ent error** is the absolute value of the error divided by the accepted e, multiplied by 100%.

$$\text{Percent error} = \frac{|\text{error}|}{\text{accepted value}} \times 100\%$$

g the absolute value of the error means that the percent error will ys be a positive value. For the boiling-point measurement, the percent is calculated as follows.

$$\begin{aligned}\text{Percent error} &= \frac{|99.1\ ^\circ\text{C} - 100.0\ ^\circ\text{C}|}{100.0\ ^\circ\text{C}} \times 100\% \\ &= \frac{0.9\ \cancel{^\circ\text{C}}}{100.0\ \cancel{^\circ\text{C}}} \times 100\% \\ &= 0.009 \times 100\% \\ &= 0.9\%\end{aligned}$$

2 Teach

ACTIVITY

Place a set of small objects near a triple-beam balance. Set a deadline by which each student will have determined the mass of each object. After everyone has had an opportunity, have students compile a summary of all the measurements for each object. Use the summary to illustrate the concept of precision.

To illustrate accuracy, determine the mass of each object on a high quality balance; compare these values to the values found by the students. To further illustrate the effect of equipment on accuracy, pour about 20 mL of water into a graduated beaker and about 20 mL into a buret; then have students take volume readings.

Significant Figures in Measurements

If you use a liquid-filled thermometer that is calibrated in 1 °C intervals, you can easily read the temperature to the nearest degree. With such a thermometer, however, you can also estimate the temperature to about the nearest tenth of a degree by noting the closeness of the liquid inside to the calibrations. Suppose you estimate a temperature that lies between 23 °C and 24 °C to be 24.3 °C. This estimated number has three digits. The first two digits (2 and 4) are known with certainty. But the rightmost digit has been estimated and involves some uncertainty. These reported digits all convey useful information, however, and are called significant figures. The **significant figures** in a measurement include all of the digits that are known, plus a last digit that is estimated. Measurements must always be reported to the correct number of significant figures because, as you will soon learn, calculated answers depend upon the number of significant figures in the values used in the calculation.

Suppose you take someone's temperature with a thermometer that is calibrated in 0.1 °C intervals. You can read the temperature with virtual certainty to the nearest 0.1 °C, and you can estimate it to the nearest 0.01 °C. You might report the temperature as 35.82 °C. This measurement has four significant figures, the rightmost of which is uncertain. Instruments differ in the number of significant figures that can be obtained from them and thus in the precision of measurements. The three meter sticks in **Figure 3.6** can be used to make successively more precise measurements of the board.

To determine whether a digit in a measured value is significant, you need to apply the following rules.

1. Every nonzero digit in a reported measurement is assumed to be significant. The measurements 24.7 meters, 0.743 meter, and 714 meters each express a measure of length to three significant figures.

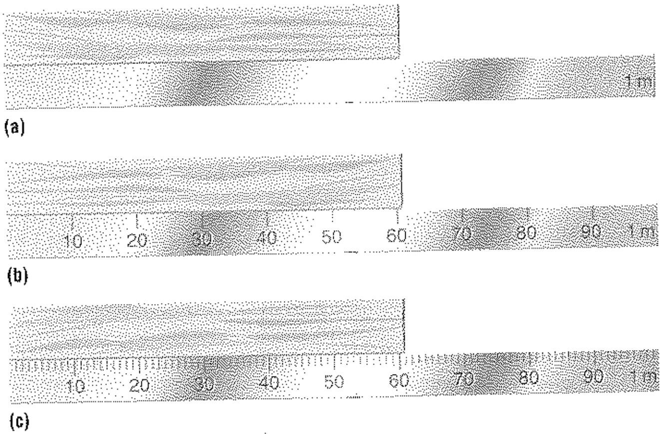


Figure 3.6

Three differently calibrated meter sticks can be used to measure the length of a board. What measurement is obtained in each case? Are there differences in the number of significant figures in the three measurements? Explain. ①

Link to

ENGINEERING

Computer-Aided Design

Measurement is important, but making or specifying a large number of measurements can be very time consuming. An engineer designing a complex machine must specify hundreds of dimensions. Today, computer-aided design (CAD) programs have replaced many tedious aspects of technical design. A CAD program usually permits a designer to begin with a few important elements and measurements of the object being designed. As the design progresses, the program computes the dimensions of added elements and produces drawings of the object from any perspective the designer desires. When one dimension in a design is changed, CAD programs automatically adjust all of the other dimensions in proportion. The introduction of CAD programs has reduced the time needed to design complex items as well as the number of errors that plague big projects.

Chem ASAP!

Animation 2

See how the accuracy of a calculated result depends on the sensitivity of the measuring instruments.

2. Zeros appearing between nonzero digits are significant. The measurements 7003 meters, 40.79 meters, and 1.503 meters each have four significant figures.

3. Leftmost zeros appearing in front of nonzero digits are not significant. They act as placeholders. The measurements 0.0071 meter, 0.42 meter, and 0.000 099 meter each have only two significant figures. The zeros to the left are not significant. By writing the measurements in scientific notation, you can get rid of such placeholding zeros: in this case, 7.1×10^{-3} meter, 4.2×10^{-1} meter, and 9.9×10^{-5} meter.

4. Zeros at the end of a number and to the right of a decimal point are always significant. The measurements 43.00 meters, 1.010 meters, and 9.000 meters each have four significant figures.

5. Zeros at the rightmost end of a measurement that lie to the left of an understood decimal point are not significant if they serve as placeholders to show the magnitude of the number. The zeros in the measurements 300 meters, 7000 meters, and 27 210 meters are not significant. The numbers of significant figures in these values are one, one, and four, respectively. If such zeros were known measured values, however, then they would be significant. For example, if the value of 300 meters resulted from a careful measurement rather than a rough, rounded measurement, the zeros would be significant. Ambiguity is avoided if measurements are written in scientific notation. For example, if all of the zeros in the measurement 300 meters were significant, writing the value as 3.00×10^2 meters makes it clear that these zeros are significant.

6. There are two situations in which measurements have an unlimited number of significant figures. The first involves counting. If you carefully count that there are 23 people in your classroom, then there are exactly 23 people, not 22.9 or 23.1. This measurement can only be a whole number and has an unlimited number of significant figures, in the form of zeros understood to be to the right of the decimal point; thus, 23.000 000 ... is understood. The second situation of unlimited significant figures involves exactly defined quantities, such as those usually used within a system of measurement. When, for example, you write 60 minutes = 1 hour, these quantities have an unlimited number of significant figures; there are exactly 60 minutes in an hour, by definition. It is important to recognize when quantities are exact and to round calculated answers correctly in problems involving such values.

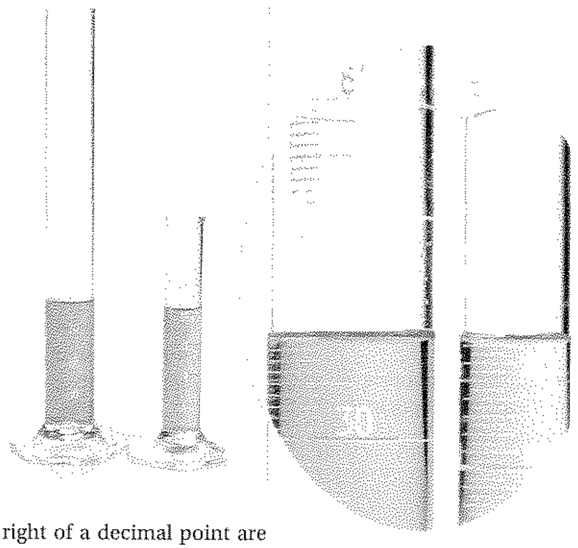


Figure 3.7
Two differently calibrated graduated cylinders are used to measure the volume of a liquid. Which cylinder would give more precise measurements? ②

Sample Problem 3-1

An engineer made the following measurements. How many significant figures are in each measurement?

- **a.** 123 meters
- **b.** 0.123 meter
- **c.** 40 506 meters
- **d.** 9.8000×10^4 meters
- **e.** 30.0 meters
- **f.** 22 meter sticks
- **g.** 0.070 80 meter
- **h.** 98 000 meters

1. *ANALYZE* ***Plan a problem-solving strategy.***
The location of each zero in the measurement and the location of the decimal point determine which of the rules apply for determining significant figures. These locations are known by examining each measurement value.

2. *SOLVE* ***Apply the problem-solving strategy.***
Examine each measurement and apply the rules for determining significant figures. All nonzero digits are significant (rule 1). Use rules 2–5 to determine if the zeros are significant.

- **a.** 3 (rule 1)
- **b.** 3 (rule 3)
- **c.** 5 (rule 2)
- **d.** 5 (rule 4)
- **e.** 3 (rule 4)
- **f.** unlimited (rule 6)
- **g.** 4 (rules 2, 3, 4)
- **h.** 2 (rule 5)

3. *EVALUATE* ***Do the results make sense?***
The rules for determining significant digits have been correctly applied in each case.

Practice Problems

5. Determine the number of significant figures in each measurement.
- **a.** 0.057 30 meter
- **b.** 8765 meters
- **c.** 0.000 73 meter
- **d.** 40.007 meters

6. How many significant figures are in each measurement?
- **a.** 143 grams
- **b.** 0.074 meter
- **c.** 8.750×10^{-2} gram
- **d.** 1.072 meters

Significant Figures in Calculations

Rounding Suppose you use a calculator to find the area of a floor that measures 7.7 meters by 5.4 meters. The calculator would give an answer of 41.58 square meters. The calculated area is expressed to four significant figures. However, each of the measurements used in the calculation is expressed to only two significant figures. It is important to know that the calculated area cannot be more precise than the measured values used to obtain it.

The calculated area must be rounded to make it consistent with the measurements from which it was calculated. In general, an answer cannot be more precise than the least precise measurement from which it was calculated.

To round a number, you must first decide how many significant figures the answer should have. This decision depends on the given measurements and on the mathematical process used to arrive at the answer. Once you know the number of significant figures your answer should have, round to that many digits, counting from the left. If the digit immediately to the right of the last significant digit is less than 5, it is simply dropped and the value of the last significant digit stays the same. If the digit in question is 5 or greater, the value of the digit in the last significant

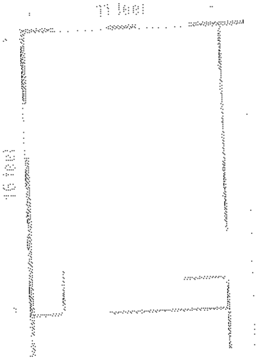


Figure 3.8

The room shown in the blueprint measures 11 feet by 16 feet. What is the calculated area of the room to the correct number of significant figures? ①

place is increased by 1. For example, rounding 56.312 meters to four significant figures produces the result 56.31 meters because 2, the digit to the right of the last significant digit, is less than 5. Rounding 56.316 meters gives the result 56.32 meters because 6, the digit to the right of the last significant figure, is greater than 5.

Sample Problem 3-2

Round each measurement to the number of significant figures shown in parentheses. Write the answers in scientific notation.

a. 314.721 meters (4)
b. 0.001 775 meter (2)
c. 64.32×10^{-1} meters (1) 6
d. 8792 meters (2) 80, 60

1. ANALYZE *Plan a problem-solving strategy.*
Using the rules for determining significant figures, round each number. Then, apply the rules for expressing numbers in scientific notation.

2. SOLVE *Apply the problem-solving strategy.*
Count from the left and apply the rule to the digit immediately to the right of the digit to which you are rounding. The arrows point to the digit immediately following the last significant digit. (The number of significant figures for each is shown in parentheses.)

a. $314.7\underset{\uparrow}{2}1$ meters

2 is less than 5, so do not round up.
314.7 meters (4) = 3.147×10^{2} meters

b. $0.001\,7\underset{\uparrow}{7}5$ meter

7 is greater than 5, so round up.
0.0018 meter (2) = 1.8×10^{-3} meter

c. $6\underset{\uparrow}{4}.32 \times 10^{-1}$ meters

4 is less than 5, so do not round up.
60×10^{-1} meters (1) = 6 meters

(Note that 6 meters can be expressed in scientific notation as 6×10^{0} meters.)

d. $87\underset{\uparrow}{9}2$ meters

9 is greater than 5, so round up.
8800 meters (2) = 8.8×10^{3} meters

3. EVALUATE *Do the results make sense?*
The rules for rounding and for writing numbers in scientific notation have been correctly applied.

Practice Problems

7. Round each measurement to three significant figures. Write your answers in scientific notation.
a. 87.073 meters
b. 4.3621×10^{8} meters
c. 0.015 52 meter
d. 9009 meters
e. 1.7777×10^{-3} meter
f. 629.55 meters

8. Round each measurement in Practice Problem 7 to one significant figure. Write your answers in scientific notation.

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Problem-Solving 7
Solve Problem 7 with the help of an interactive guided tutorial.

Addition and Subtraction The answer to an addition or subtraction calculation should be rounded to the same number of decimal places (not digits) as the measurement with the least number of decimal places. Work through Sample Problem 3-3 below which provides examples of rounding in addition and subtraction calculations.

Sample Problem 3-3

Perform the following addition and subtraction operations. Give each answer to the correct number of significant figures.

a. 12.52 meters + 349.0 meters + 8.24 meters

b. 74.626 meters − 28.34 meters

1. ***ANALYZE* *Plan a problem-solving strategy.***
   Perform the required math operation and then analyze each measurement to determine the number of decimal places required in the answer.

2. ***SOLVE* *Apply the problem-solving strategy.***
   Round the answers to match the measurement with the least number of decimal places.

   a. Align the decimal points and add the numbers.

$$\begin{array}{rl} 12.52 & \text{meters} \\ 349.0 & \text{meters} \\ +\ \ 8.24 & \text{meters} \\ \hline 369.76 & \text{meters} \end{array}$$

   The second measurement (349.0 meters) has the least number of digits (one) to the right of the decimal point. Thus the answer must be rounded to one digit after the decimal point. The answer is rounded to 369.8 meters, or 3.698×10^2 meters.

   b. Align the decimal points and subtract the numbers.

$$\begin{array}{rl} 74.626 & \text{meters} \\ -\ 28.34 & \text{meters} \\ \hline 46.286 & \text{meters} \end{array}$$

   The answer must be rounded to two digits after the decimal point to match the second measurement. The answer is 46.29 meters, or 4.629×10^1 meters.

3. ***EVALUATE* *Do the results make sense?***
   The mathematical operations have been correctly carried out and the resulting answers are reported to the correct number of decimal places.

Practice Problems

9. Perform each operation. Give your answers to the correct number of significant figures.
  - **a.** 61.2 meters + 9.35 meters + 8.6 meters
  - **b.** 9.44 meters − 2.11 meters
  - **c.** 1.36 meters + 10.17 meters
  - **d.** 34.61 meters − 17.3 meters

10. Find the total mass of three diamonds that have masses of 14.2 grams, 8.73 grams, and 0.912 gram.

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Problem-Solving 10

Solve Problem 10 with the help of an interactive guided tutorial.

Multiplication and Division In calculations involving multiplication and division, you need to round the answer to the same number of significant figures as the measurement with the least number of significant figures.

You can see in **Figure 3.9** that the calculator answer (5.7672) must be rounded to three significant figures because each measurement used in the calculation has only three significant figures.

The position of the decimal point has nothing to do with the rounding process when multiplying and dividing measurements. The position of the decimal point is important only in rounding the answers of addition or subtraction problems.

Figure 3.9

This calculator was used to multiply the length and width measurements of a bolt of fabric, 3.24 meters by 1.78 meters, each of which has three significant figures. The area of the fabric is really not known with the precision suggested by the calculator. What is the product when correctly rounded? ①

Sample Problem 3-4

Perform the following operations. Give the answers to the correct number of significant figures.

a. 7.55 meters $\times$ 0.34 meter
b. 2.10 meters $\times$ 0.70 meter
c. 2.4526 meters $\div$ 8.4
d. 0.365 meter $\div$ 0.0200

1. *ANALYZE* ***Plan a problem-solving strategy.***
Perform the required math operation and then analyze each of the original numbers to determine the correct number of significant figures required in the answer.

2. *SOLVE* ***Apply the problem-solving strategy.***

Round the answers to match the measurement with the least number of significant figures.

a. 7.55 meters $\times$ 0.34 meter = 2.567 square meters = 2.6 square meters
(0.34 meter has two significant figures.)

b. 2.10 meters $\times$ 0.70 meter = 1.47 square meters = 1.5 square meters
(0.70 meter has two significant figures.)

c. 2.4526 meters $\div$ 8.4 = 0.291 976 meter = 0.29 meter
(8.4 has two significant figures.)

d. 0.365 meter $\div$ 0.0200 = 18.25 meters = 18.3 meters
(Both numbers have three significant figures.)

3. *EVALUATE* ***Do the results make sense?***
The mathematical operations have been performed correctly, and the resulting answers are reported to the correct number of places.

Practice Problems

11. Solve each problem. Give your answers to the correct number of significant figures and in scientific notation.
a. 8.3 meters $\times$ 2.22 meters
b. 8432 meters $\div$ 12.5
c. 35.2 seconds $\times$ 1 minute/60 seconds

12. Calculate the volume of a warehouse that has inside dimensions of 22.4 meters by 11.3 meters by 5.2 meters. (Volume = $l \times w \times h$)

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Problem-Solving 12
Solve Problem 12 with the help of an interactive guided tutorial.

MINI LAB

Accuracy and Precision

PURPOSE

To measure the dimensions of an object as accurately and precisely as possible and to apply rules for rounding answers calculated from the measurements.

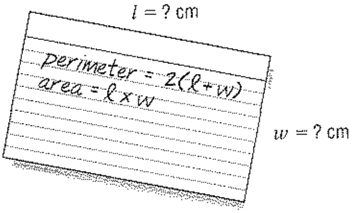
MATERIALS

- index card (3″ × 5″)
- metric ruler

PROCEDURE

1. Use a metric ruler to measure in centimeters the length and width of an index card as accurately and precisely as you can. The hundredths place in your measurement should be estimated.
2. Calculate the perimeter [2 × (length + width)] and the area (length × width) of the index card. Write both your unrounded answers and your correctly rounded answers on the chalkboard.

ANALYSIS AND CONCLUSIONS



1. How many significant figures are in your measurements of length and of width?
2. How do your measurements compare with those of your classmates?
3. How many significant figures are in your calculated value for the area? In your calculated value for the perimeter? Do your rounded answers have as many significant figures as your classmates' measurements?
4. Assume that the correct (accurate) length and width of the card are 12.70 cm and 7.62 cm, respectively. Calculate the percent error for each of your two measurements.

section review 3.2

13. Explain the differences between accuracy, precision, and error of a measurement.

14. Determine the number of significant figures in each of the following measurements and calculation results.

a. 12 basketball players
b. 0.010 square meter
c. 507 thumbtacks
d. 0.070 020 meter
e. 10 800 meters
f. 5.00 cubic meters

15. Solve the following and express each answer in scientific notation.

a. $(5.3 \times 10^{4}) + (1.3 \times 10^{4})$
b. $(7.2 \times 10^{-4}) \div (1.8 \times 10^{3})$
c. $10^{4} \times 10^{-3} \times 10^{6}$
d. $(9.12 \times 10^{-1}) - (4.7 \times 10^{-2})$
e. $(5.4 \times 10^{4}) \times (3.5 \times 10^{9})$
f. $(1.2 \times 10^{2}) \times (8.9 \times 10^{2})$

16. A technician experimentally determined the boiling point of octane to be 124.1 °C. The actual boiling point of octane is 125.7 °C. Calculate the error and the percent error.

Chem ASAP! **Assessment 3.2** Check your understanding of the important ideas and concepts in Section 3.2.

portfolio project

Interview workers who construct buildings or highways about the importance of accurate measurements.

MINI LAB

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PURPOSE

To measure the dimensions of an object as accurately and precisely as possible and to apply rules for rounding answers calculated from the measurements.

MATERIALS

- index card (3″ × 5″)
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PROCEDURE

1. Use a metric ruler to measure in centimeters the length and width of an index card as accurately and precisely as you can. The hundredths place in your measurement should be estimated.
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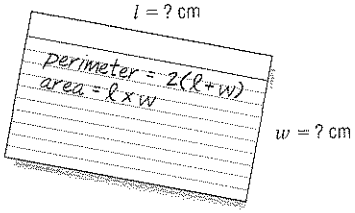

ANALYSIS AND CONCLUSIONS

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b. $(7.2 \times 10^{-4}) \div (1.8 \times 10^3)$
c. $10^4 \times 10^{-3} \times 10^6$
d. $(9.12 \times 10^{-1}) - (4.7 \times 10^{-2})$
e. $(5.4 \times 10^4) \times (3.5 \times 10^9)$
f. $(1.2 \times 10^2) \times (8.9 \times 10^2)$

16. A technician experimentally determined the boiling point of octane to be 124.1 °C. The actual boiling point of octane is 125.7 °C. Calculate the error and the percent error.

Chem ASAP! **Assessment 3.2** Check your understanding of the important ideas and concepts in Section 3.2.